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S.I.: PLURALISTIC PERSPECTIVES ON LOGIC

A note on mathematical pluralism and logical pluralism

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Abstract

Mathematical pluralism notes that there are many different kinds of pure mathematical structures—notably those based on different logics—and that, *qua* pieces of pure mathematics, they are all equally good. Logical pluralism is the view that there are different logics (consequence relations), which are, in an appropriate sense, equally good. Some, such as Shapiro (Varieties of logic, Oxford University Press, Oxford, 2014), have argued that mathematical pluralism entails logical pluralism. In this brief note I argue that this does not follow. There is a crucial distinction to be drawn between the preservation of truth (*simpliciter*) and the preservation of truth-in-a-structure; and once this distinction is drawn, this suffices to block the argument. The paper starts by clarifying the relevant notions of mathematical and logical pluralism. It then explains why the argument from the first to the second does not follow. A final section considers a few objections.

Keywords Mathematical pluralism · Logical pluralism · Truth · Truth in a structure · Intuitionist logic · Paraconsistent logic

1 Introduction

Mathematical pluralism, in the sense that I will understand it,[1] is the view that there are different pure mathematics—notably those based on different logics—that are, in an appropriate sense, equally good. Logical pluralism, again in the sense that I will understand it, is the view that there are different logics (consequence relations), which

[1] As does Shaprio (2014).

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are, in an appropriate sense, equally good.[2] Mathematical pluralism might naturally be thought to entail logical pluralism. In this brief note, I will explain why it does not. The conjunction of mathematical pluralism and logical monism is a perfectly coherent position.

2 Mathematical pluralism

Let us start with mathematical pluralism.[3] A fairly orthodox assumption of the philosophy of mathematics in the 20th century was that reasoning in mathematics was to be pursued in classical (aka Frege/Russell) logic. It now appears, however, that there are perfectly legitimate mathematical theories in which this is not, indeed cannot, be the case.

Thus, take intuitionist logic. Intuitionist theories of the natural numbers, the reals, and so on, are well known.[4] Moreover, there are important cases where imposing classical logic on the theory does not simply collapse the theory into the corresponding classical one, but results in total collapse (triviality).

Let me give just one example of this. This is the Kock–Lawvere theory of smooth infinitesimal analysis.[5] To motivate this, consider an informal way in which one might compute the derivative of a function, $f(x)$, using infinitesimals.[6] The derivative, $f'(x)$, is the slope of the function at x, given an infinitesimal displacement, i; so $f(x+i) - f(x) = if'(x)$. Now, as an example, take $f(x)$ to be x^3. Then $if'(x) = (x+i)^3 - x^3 = 3x^2i + 3xi^2 + i^3$. If we could divide by i, we would have $3x^2 + 3xi + i^2$. Setting i to 0 delivers the result—though how, then, did we divide by i?

The theory of smooth infinitesimals addresses this matter. If $i^2 = 0$, we have another route to the answer. For then it follows that for any infinitesimal, i, $if'(x) = 3x^2i$. We may not be able to divide by i, but suppose that $ai = bi$, for all i, implies that $a = b$ (this is the *Principle of Microcancellation*). $f'(x) = 3x^2$ then follows. This is exactly how the theory of smooth infinitesimal analysis proceeds.

Call a real number, i, a *nilsquare* if $i^2 = 0$. Of course, 0 itself is a nilsquare, but it may not be the only one! We may think of the nilsquares as infinitesimals. The theory of smooth infinitesimals takes functions to be linear on these. Given a function, f, there is a unique r such that, for every nilsquare, i, $f(x+i) - f(x) = ri$ (In effect, r is the derivative of f at x.). This is the *Principle of Microaffineness*.[7]

Microaffineness implies that 0 is not the only nilsquare. For suppose that it is, then all we have is that $f(x+0) - f(x) = r0$, and clearly this does not define a unique r. So:

[2] The term 'logic' is highly ambiguous. For the relevant sense in the case of logical pluralism, see Priest (2006). Further discussion of the ambiguity of the term can be found in Priest (2014).

[3] Mathematical pluralism is defended at length in Priest (2013), (2019) and, especially, Shapiro (2014).

[4] On intuitionist mathematics, see Dummett (2000).

[5] On which, see Bell (2008).

[6] A way that is very close to how eighteenth century mathematicians actually proceeded. See Brown and Priest (2004) and Sweeney (2014).

[7] Microcancellation follows. Take $f(x)$ to be xa. Then, taking x to be 0, Microaffineness implies that there is a unique r such that, for all i, $ai = ri$. So if $ai = bi$ for all i, $a = r = b$.

[1] $\neg\forall i(i^2 = 0 \rightarrow i = 0)$

But now, why do we need intuitionist logic? Well, one might argue that 0 *is* the only nilsquare, which would make a mess of things. A typical piece of reasoning for this goes as follows. Suppose that i is a nilsquare and that $\neg i = 0$. Then i has an inverse, i^{-1}, such that $i \cdot i^{-1} = 1$. But then $i^2 \cdot i^{-1} = i$. Since $i^2 = 0$, it follows that $i = 0$. Hence, by *reductio*, we have shown that $\neg\neg i = 0$. If we were allowed to apply Double Negation, we could infer that $i = 0$—and so we would have a contradiction on our hands. But this move is not legitimate in intuitionist logic. We have just:

[2] $\forall i(i^2 = 0 \rightarrow \neg\neg i = 0)$

And given intuitionistic logic, we may hold both [1] and [2] together.

For a second example of this phenomenon, we may turn to paraconsistent logic. There are inconsistent theories of clear mathematical interest based on such a logic. Such theories include theories of arithmetic, set theory, and a number of other topics.[8] And here the imposition of classical logic on the theory always results in total collapse.

Again, let me give one of the perhaps lesser known examples. This concerns boundaries. Take a simple topological space, say the one-dimensional real line. Divide it into two disjoint parts, left, L, and right, R. Now consider the point of division, p. Is p in L or R? Of course, the description under-determines an answer to the question. But when the example is fleshed out, considerations of symmetry might suggest that it is in both. Then, $p \in L$, and $p \in R$ so $p \notin L$—and symmetrically for R. So a description of the space might be that if $x < p$, x is (consistently) in L; if $p < x$, p is (consistently) in R; and p is both in and not in L, and in and not in R. Given an appropriate paraconsistent logic, the description is quite coherent.

This might not seem particularly interesting, but the idea of inconsistent boundaries has interesting applications. One of these is to describe the geometry of "impossible pictures", that is, representations of objects that are impossible in Euclidean 3-space.[9] Consider the following picture:

The content of the picture is impossible. How should one describe it mathematically? Any mathematical characterisation will specify, amongst other things, the orientations of the various faces. Now, consider the left-hand face, and in particular its lighter shaded part. This is 90° to the horizontal. Next, consider the top of the lower step on the right-hand side of the picture. This is 0° to the horizontal. Finally, consider the boundary between them (a vertical line on the diagram). This is on both planes. Hence it is at both 90° and 0° to the horizontal.[10] That's a contradiction, since it cannot be

[8] For an overview, see Mortensen (2017).

[9] For more on the following, see Mortensen (2010).

[10] And one can set things up in such a way that this does not imply that $90 = 0$.

both; but that's exactly what makes the content of the picture impossible. Note that the characterisation of the content must deploy a paraconsistent logic, since it should not imply, e.g., that the top of the higher step is at 90° to the horizontal.

What we see, then, is that there are very distinctive fields of intuitionistic and paraconsistent mathematics, quite different from the fields of classical mathematics. Moreover, one does not have to think that intuitionism or paraconsistency is *philosophically* correct, or that the mathematical theories are true (whatever one might take this to mean), to recognise that these are interesting mathematical enterprises with their own integrity.[11] They are perfectly good parts of pure mathematics.[12]

Thus, there is a plurality of pure mathematical investigations such that none of them can be reduced to the others—in the sense in which it is standardly assumed that all mathematics can be reduced to ZFC. This is mathematical pluralism. In truth, this pluralism was already clear in the case of set theory and category theory, even though these are standardly taken to have the same underlying logic: attempts to reduce either to the other, were always straining at the seams.[13] But intuitionist and paraconsistent mathematics appear to have put the matter beyond doubt. We have, in these cases, mathematical enterprises which are completely *sui generis*. Note that this does not imply that all these branches of mathematics are equally deep, rich, elegant, applicable, etc. That is a quite different matter. The point is just that they are all equally legitimate pure mathematical structures.[14]

3 Logical pluralism

So let us turn to logical pluralism; and let us start by getting some conceptual geography straight.[15]

Distinguish first between a pure logic and an applied logic. Consider, for analogy, geometry. There are many pure geometries: Euclidean, hyperbolic, elliptic, etc. These are all perfectly good pure mathematical structures; and there is no sense in which any

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[11] Which is not to say that all mathematics *are* interested in them. Most mathematicians are interested in only parts of mathematics. It is to say that these theories have a mathematical structure which warrants mathematical interest, and so interests some mathematicians.

[12] A referee of a previous draft objected that this begs the question against a mathematical monist. These theories are not legitimate. The institution of mathematics is just making a mistake about what pure mathematics really is. This is actually beside the point here. I am not arguing for the truth of mathematical pluralism. I am arguing that logical pluralism does not follow from it. However, I do indeed think that mathematical pluralism is true. To suggest that mathematicians are mistaken as to what is legitimate mathematics strikes me as philosophical hubris.

[13] For discussion, see Priest (2019), §8. One might also suggest that the matter was clear within set theory itself. For, given that, say, the Continuum Hypothesis (CH) is independent of ZFC, mathematicians investigate the theories ZFC+CH and ZFC+¬CH. However, in this case, one might suggest that this is not really pluralism, since both investigations can be subsumed under classical model theory. In a similar way, it is often pointed out that the internal logic of a topos is intuitionistic logic. However, this is established in standard category theory, using classical logic.

[14] I use the word 'structure' here in the way that is standard in mathematics, namely a complex of objects and the relations and functions between them. This has absolutely nothing to do with mathematical structuralism as a philosophy of mathematics, as one referee wondered.

[15] For more on this, see Priest (2006).

of them is right or wrong. But of course, geometries can be applied for many purposes: charting the spatio-temporal structure of the cosmos, geo-surveying, etc. The question of which geometry is right for a particular application then becomes a pertinent one.

So it is with logic. There are many pure logics: classical logic, intutionistic, paraconsistent, etc. These are simply pure mathematical structures: proof systems, model theories, algebras, etc. And there is no sense to the question of which is right and which is wrong. But pure logics can have applications. Indeed, they can have many applications: simplifying electrical circuits, analysing grammatical structures (the Lambek Calculus), and so on. The question of which pure logic is right for a particular application then becomes a pertinent one.

Next, geometry has what one might call a *canonical application*: characterising the structure of space. When Euclid formulated his axiomatic geometry, he was just systematising what was known about the spatial geometry of his day.[16] It turns out that this was the wrong general geometry for that application—in General Relativity, the geometry is one of variable curvature; but never mind, that was its intended application. Similarly, logic has a canonical application: evaluating the validity of arguments. Syllogistic was invented by Aristotle for this purpose, though, as with Euclidean geometry, we now think that this is not the right logic for the job.

Note that when people argue, be they lawyers, politicians, historians, scientists, or wot not, they do not do so in a formal language. This is just as true of mathematicians. If you open the pages of a mathematics journal or text book, you will not find the argument presented in *Principia*-ese, or any other formal language. People argue in a natural language (though some of the vocabulary used may be of a technical nature). And if such an argument is offered, we may legitimately ask, 'Is it valid?' Perhaps the answer will be, 'It depends what you mean'. But once this is made sufficiently clear, we would normally expect a straight answer. Someone who said, 'Well, it's valid in classical logic, but not intuitionist logic' would fairly be taken to be avoiding the question.[17] It is exactly that question to which the canonical application of logic is supposed to provide an answer.

Of course, there is more to human reasoning than deduction, as Harman (1982) noted. What he points to there is what is covered by contemporary discussions of belief revision. A theory of logical consequence (on its own) will not deliver this. But a theory of belief revision presupposes an account of deductive validity. The exact connection is a sensitive matter, but it would appear to be something like the following: if A entails B (or, maybe, if A is known to entail B), one should not accept A and reject B. Further details are not relevant here. The point is simply that evaluating claims to validity is the canonical application of logic.

These clarifications made, I can now state simply what logical pluralism is. Logical pluralism gets articulated in many different ways,[18] but the sense I am concerned with here is this: there are many pure logics whose consequence relations deliver equally good answers to that the question of the paragraph before last.

16 'Euclid's treatise contains a systematic exposition of the leading propositions of elementary metrical geometry.' Rouse Ball (1960), p. 44.

17 Though if this were a response in a philosophical debate, it might be a prelude to a discussion of which of these logics is right; but that is another matter.

18 On the variety of logical pluralisms, see Priest (2006).

4 Mathematical pluralism and logical pluralism

Now, mathematical pluralism might well be thought to deliver this logical pluralism.[19] For when we reason about ZFC sets, classical logic is the correct logic to apply; when we reason about smooth infinitesimals, intuitionistic logic is the correct logic to apply; when we reason about inconsistent topologies, paraconsistent logic is the correct logic to apply.

It must be agreed that there is something right about this view. If we are reasoning about ZFC sets, it is appropriate to use classical logic; for this structure is closed under classical consequence. If we reason about smooth infinitesimals it is appropriate to use intuitionistic logic; for this structure is closed under intuitionistic consequence. If we reason about inconsistent topologies it is appropriate to use paraconsistent logic; for these structures are closed under paraconsistent consequence. Each logic is the correct logic for preserving truth-in-the-structure. It is the internal logic of the structure, if one may put it that way.

But the canonical application of logic is not about truth-in-a-structure-preservation. It is about truth-preservation. When we reason, we are interested in whether, given that our premises are true [or assuming them to be true] our conclusion is [would be] so as well. That the canonical application of logic is about truth preservation is not a profound claim; in some sense, it is a simple truism. Of course, it is a contentious matter as to how to spell out exactly what it means.[20] Nor is it even clear what machinery is best employed to articulate the thought: proof procedures, set-theoretic interpretations, modal notions, probability theory? These matters are not pertinent here, though. The point is the simple distinction between truth-preservation, however one understands this notion, and the preservation of truth-in-a-structure. And once this distinction between the two is noted, it is clear that the fact that there are different ways to preserve truth in a structure, depending on the structure, does not imply that there are many ways to preserve truth, *simpliciter*.

One way to make the point is as follows. Mathematical theories are not exactly stories, but they are much closer than one might have thought.[21] In particular, when reasoning about a mathematical structure, $\mathfrak{A}$, we, in effect, prefix our statements with the operator 'In structure $\mathfrak{A}$...'. When we reason about what holds in a work of fiction, we, in effect, prefix our reasoning with the operator 'In fiction F...'. And in both cases, our reasoning respects the internal logic of the structure/fiction—which does not have to be classical logic.[22] Of course, traditionally, mathematicians did not think of their statements as having this prefix. But that this is so is what mathematical

[19] Indeed, Shapiro (2014) takes it to do so.

[20] One thing it cannot mean is that if $A \vdash B$ then the conditional $T\langle A\rangle \rightarrow T\langle B\rangle$ holds, where T is a naive truth predicate. For then trivialism in the shape of the Curry Paradox follows. However, there is no bar to truth preservation in the following form: if $A \vdash B$ then $T\langle A\rangle \vdash T\langle B\rangle$—or a number of others. For a general discussion of the matter, see Priest (2010), §13.

[21] See Priest (2016), §7.4.

[22] For a fiction in which the internal logic is a paraconsistent logic, see 'Sylvan's Box', Priest (2016), §6.6.

pluralism appears to have shown us.[23] And truth-in-a-structure is not a species of truth *simplicter*—any more than truth in a fiction is.

Nor does the fact that in applied mathematics we aim at truth *simplicter* change matters. We are talking about pure mathematics here. When we apply a piece of pure mathematics (be it in physics, economics, biology, or whatever), the pure mathematical statements become (with luck!) empirical truths about the domain of application.[24] The question then, is what pure mathematical structure does this. In other words, which bit of pure mathematics is the right pure mathematics for the job. That is a different matter.[25]

Now, I have drawn a distinction between being pure and being applied in both logic and mathematics. Yet the version of pluralism in mathematics I am considering is with respect to pure mathematics, and the version of logical pluralism I am considering is with respect to applied logic. It might be suggested that it would be fairer to compare like with like. But there are, in fact, good reasons for proceeding in the way I do. First, those who have defended mathematical pluralism (see fn 3) have defended it with respect to pure mathematics. And Shapiro at least takes this to imply logical pluralism in my sense. Moreover, there are good reasons for being concerned with pluralism with respect to applied logic. Pluralism with respect to pure logic is a no-brainer. It needs no support from mathematical pluralism. Indeed, logical pluralism with respect to different applications is also a no-brainer. Boolean logic is clearly the correct pure logic to apply for simplifying electronic circuits. The substructural logic which is the Lambek Calculus is the correct pure logic to apply for grammatical parsing. So the interesting question is whether pluralism about pure mathematics entails pluralism about applied logic with its canonical application.[26]

5 Interlude: fragmenting truth

At this point, one might be inclined to reply that there might well be a pluralism of truths, and so a pluralism of preservations-of-truth.

One way to argue for this would be to be a relativist about truth. Thus, one might suppose that there are many equally good truths, e.g., Christian, Moslem, Buddhist. I

[23] I note that there are versions of fictionalism about mathematics which treat mathematical claims as tacitly prefixed by an operator such as ‘According to the fiction F...’ (See Balaguer (2011) and Leng (2018).). This kind of fictionalist might well think of the prefix ‘In structure $\mathfrak{A}$...’ as ‘In the fiction F...’. The account given here should, then, be very agreeable to this kind of mathematical fictionalist. However it is not committed to such fictionalism.

[24] On applied mathematics, see Priest (2016), 7.8.

[25] As a referee noted, in his (2018) Williamson argues that applied mathematics poses a challenge for non-classical logic. This is not the place to discuss the cogency of his argument; it is irrelevant to the position being argued here, which is quite compatible with there being one true logic, and that logic being classical.

[26] If there were a pluralism in applied logic, this might arguably imply logical pluralism of the kind in question. However, at least so far, the parts of mathematics that get applied (number theory, analysis, various geometries, probability theory, etc.) can be seen, in the usual way, as fragments of classical ZFC. So there is as yet no case to be made for logical pluralism here.

would certainly reject such relativism.[27] Moreover, I hardly think that many of those who are interested in this issue would want to buy into such post-modernism.

Another suggestion concerns so called truth pluralism.[28] Put simply, truth pluralism is the view that different kinds of discourse have different kinds of truth-makers. There is, yet, still a single notion of truth *simpiciter*.

A third suggestion is as follows. There are certain logical pluralists, such as da Costa and those who follow him, who argue that different subjects about which we reason require different consequence relations. Reality itself is, as it were, fragmented. And does not a fragmentation of reality imply a fragmentation of truth? No, it entails a truth pluralism, which is a different matter, as I have just noted. To see this, suppose, for example, that reality is fragmented into the macro and the micro physical levels. Note that we need to be able to reasons about both of these at the same time—for example, concerning the interaction between entities of the two domains. What logic, i.e., machinery of truth preservation, do we use? Presumably the logic which is the intersection of logics for the two fragments. There is, hence, a single notion of truth-preservation. Of course, it can remain the case that when the quantifiers are bounded to one or other of the domains, the appropriate logics may be employed, due to the addition of extra "contingent" principles concerning the domain in question.[29]

In any case, all the above matters are beside the point. What they all do is provide an argument for the fragmentation of truth, and so for the fragmentation of the preservation of truth, and so for logical pluralism. Whether or not they succeed, none of them argues that truth is fragmented because of mathematical pluralism. And the question at issue here is whether *mathematical pluralism* implies logical pluralism.

6 Objections

So let us turn to some objections that *are* relevant. These concern the very distinction between truth and truth-in-a-structure. How might one challenge this?

One way might be as follows. Truth preservation *simplicter* is validity. Validity is truth preservation in all interpretations. Interpretations are the same thing as structures. So validity is just truth preservation in all structures.

Several issues would have to be addressed to make this argument cogent. One would need to defend a model-theoretic account of validity. One would have to face the fact that in many logics validity is not defined in terms of truth preservation, but in some other terms. (For example, in many-valued logics it is defined in terms of the preservation of designated values.) And one would have to defend the anything but obvious claim that structures and interpretations are the same thing.

But even assuming that these points can be adequately addressed, there is a simple and obvious problem with the objection. Model-theoretic validity is not truth-preservation in all interpretations. Different logics (intuitionist, classical, para-

27 It has always seemed to me that most of those who endorse such a view simply confuse what is true and what is held to be true.

28 On truth pluralism, see Pederson and Wright (2013).

29 See Priest (2006).

consistent, etc) have different *kinds* of interpretations. Their model theories therefore provide an understanding of truth preservation in the appropriate kind of structure—the internal logic of the structure—not validity *simpliciter*. And if one really defines validity as truth-preservation in *all* interpretations then, given the plurality of formal logics on which mathematical structures may be based, the logic will amount, as near as makes no difference, to the null logic: no inference is valid. Such would clearly make validity useless for evaluating the validity ordinary arguments, and so cannot be right.

A second way to challenge the distinction between truth and truth-in-a-structure might be as follows. That validity is about truth preservation is, in the sense we have been dealing with, a banal claim. It is an equally banal claim that truth, whatever it is, is determined by reality. And isn't reality just another structure? So reasoning about truth is simply a variety of reasoning about truth-in-a-structure. In some sense, I suppose, reality is a structure, or at least, has a structure. But it's not any old structure. It is highly privileged. I ask you whether it is true that there were more people at Trump's presidential inauguration than at Obama's. I don't need to tell you that I want an answer that corresponds to reality.

One way to see the point is this. We have both been reading Conan Doyle's *The Hound of the Baskervilles*. We argue about whether Holmes used a Colt revolver. I say he did. You say he used a Smith and Wesson. Of course, neither of us thinks that our claims are literally true: Doyle's text is just a fiction. We are both tacitly prefixing our claims with 'In *The Hound of the Baskervilles*...'. We just omit this because, given the mutually understood context, it is unnecessary.

Now, by contrast, if I ask you whether Trump's crowd was bigger than Obama's, and you say 'No, the crowd was much smaller', I don't have to understand you as saying 'In reality, the crowd was much smaller'; and if you did, the prefix would be entirely otiose. Reality, then, is not simply a structure, on a par with other structures.

7 Conclusion

In this essay, I have not argued *against* logical pluralism, merely that it does not follow from mathematical pluralism. Nor have I argued *for* logical monism, much less that any one logic is the correct logic. All I have done is show why a mathematical pluralist may yet be a logical monist. The distinction between truth and truth in a structure lies between these two things.[30]

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